# EQUATION OF STATE FOR A BOILING LIQUID

V. F. Prisnyakov

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Universal dimensionless-pressure functions are found which can be used to estimate the error involved in approximating the equation of state as a linear dependence of the density of the liquid–vapor mixture on its enthalpy.

In studies of processes in steam generators, the equation of state of the boiling liquid is sometimes represented as a linear dependence of the specific volume v on the enthalpy [1, 2]:

$$v = a + bi, \tag{1}$$

where the coefficients $a$ and b are assumed to depend weakly on the pressure (or the boiling temperature). Let us determine the form of these dependences; if $v = v' + x(v'' - v')$ and $i = i' + xr$, we have

$$a = v' - \frac{v'' - v'}{r} i', \tag{2}$$

$$b = \frac{v'' - v'}{r}. \tag{3}$$

It is sufficiently accurate to assume that the physical parameters of the liquid and vapor on the saturation curve, i.e., the liquid enthalpy i, the specific volumes v' and v'' of the liquid and vapor, and the heat of vaporization r, are single-valued functions of only the pressure [3] or only the temperature [4]. The value of any parameter can be represented as the product of some constant for the given liquid and parameter and a function (universal for a group of thermodynamically similar substances) of the dimensionless temperature $\overline{T} = T/T_{cr}$ or the dimensionless pressure $\overline{p} = p/p_{cr}$ ($T_{cr}$ and $p_{cr}$ are the critical values):

$$\chi = \chi_* \pi_\chi.$$

Graphs of the universal function $\pi_\chi$ and the values of the constant coefficients $\chi_*$ for several thermodynamically similar substances can be found in the articles by Borishanskii [3] and Povarin [4]. Approximation of these functions by analytic expressions yields:

$$\pi_v' = v_1 + v_2 \overline{T},$$

$$\pi_v'' - \pi_v' = \pi_v''' = v_3 \overline{T}^n,$$

$$\pi_r = r_1 - r_2 \overline{T}^2,$$

$$\pi_i' = i_1 \overline{T} + i_2 \overline{T}^2.$$

Here we have arbitrarily equated the initial enthalpy to zero; this has no fundamental bearing on this discussion. The constant coefficients in these expressions for a group of substances thermodynamically similar to water (see [3, 4] for a definition of these substances) are: $v_1 = 0.03968$; $v_2 = 0.08445$; $v_3 = 0.55$; $n = -10.075$; $r_1 = 4.85$; $r_2 = 3.27$; $i_1 = 3$; and $i_2 = 1.85$.

On the basis of the above discussion, we can now obtain without difficulty the following expressions for $a$ and b:

$$a = v_* \left( \pi_v' - \frac{i_*}{r_*} \frac{\pi_v''' \pi_i'}{\pi_r} \right), \tag{4}$$

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$$b = \frac{v_*}{r_*}\,\frac{\pi_v'''}{\pi_r}. \tag{5}$$

Calculations for water show that the first term in (4) has essentially no effect on $a$ up to $\overline{T} = 0.8$, and that its effect becomes significant only near the critical parameters (at $\overline{T} > 0.9$, the first term increases, exceeding 4.4%). We therefore assume that

$$a = -\frac{v_* i_*}{r_*}\,\frac{\pi_v''' \pi_i'}{\pi_r}.$$

The coefficients $a_* = v_* i_*/r_*$ and $b_* = v_*/r_*$ depend only on the nature of the substance, and can be calculated by any working medium. We can therefore find the following functions of the dimensionless parameter $\overline{p}$ (or $\overline{T}$) from (5) and (6):

$$\pi_a = -\frac{\pi_v''' \pi_i'}{\pi_r}, \tag{7}$$

$$\pi_b = \frac{\pi_v'''}{\pi_r}.$$

The calculated results show that $\pi_a$ and $\pi_b$ can be represented as power-law dependences:

$$\pi_a = a_1 \overline{p}^n,$$

$$\pi_b = b_1 \overline{p}^m,$$

where we have $a_1 = 1.7$, $n = -0.75$, $b_1 = 0.31$, and $m = -1$ for the group of substances which are thermodynamically similar to water (at pressures below 30 bars, the error in the $\pi_b$ determination of these $b_1$ and $m_1$ values increases, exceeding 4%). For the group of metals defined in [5], we have $a_1 = -0.86$, $n = -0.19$, $b_1 = 0.67$, and $m = -0.82$.

The exponents for both functions are seen to be significantly different from zero, so the assumptions that $n = -1$ and $m = -1$ give a more accurate result (see [1]).

Accordingly, we can use functions (7) to find the changes in $\pi_a$ and $\pi_b$ (which are proportional to $a$ and b) over the pressure range in a steam-generator channel, and finally estimate the error in the determination of the specific volume; for this purpose, it is more convenient to use the equation of state (1) converted to the form

$$\overline{v} = \frac{i_*}{r_*}(\pi_a + \pi_b \overline{i}),$$

where

$$\overline{v} = v/v_*, \quad \overline{i} = i/i_*.$$

## LITERATURE CITED

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